Speed of Light

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This report compares and contrasts two optical methods of determining the speed of light, c. In the first a distance is altered, and in the second, the frequency of the light. The final value from the first method was $309 \pm 3 \times 10^6$ m s^{-1}, which is in relatively poor agreement with the literature value. However, this was deemed a superior method to the second detailed in the report, which produced a value of $193 \pm 7 \times 10^6$ m s^{-1}.

1. INTRODUCTION

The determination of the speed of light (c) has historically been a struggle due to its large magnitude in comparison to speeds encountered in daily life. Both astronomical and optical techniques have been employed to attempt to find a numerical value through history, but here we will be using two optical methods. Both methods rely on a relation between the path difference (Δ) and the phase difference (φ) of two waves, detected at a time t of,

$$\frac{\Delta}{\lambda} = \frac{\varphi}{2\pi}, \tag{1}$$

where λ is the wavelength of the waves, which normalises the left hand side of (1) to be equal to the right hand side, as 2π is the maximum phase difference that can be observed [1]. In this report we will consider two methods which rely on this relation: one in which we alter the path difference and measure the phase difference (**Method 1**); and another in which we alter the frequency of emitted light and again measure the phase difference (**Method 2**).

In **Method 1**, the path difference can be split into the sum of two distances. One we will keep constant, and one we will alter, which we will call d and $2L$, respectively. We have decided to use $2L$ rather than L since we have a track of length L which the light will travel down, and back up again, hence travelling a total distance of $2L$. For light of frequency f, and using the relation $c = f\lambda$, equation (1) can be written,

$$\varphi = \frac{2\pi f d}{c} + \frac{4\pi f}{c}L. \tag{2}$$

In **Method 2**, the frequency f is altered and the path difference $\Delta = d+2L$ is constant. Therefore, unlike **Method 1**, d cannot be neglected and must be measured to some precision. Equation (1) becomes,

$$\varphi = \frac{2\pi\Delta}{c}f. \tag{3}$$

2. METHODS

An oscilloscope took two inputs: one from a laser emitter, which took an input from a signal generator capable of a range of frequencies up to 100 MHz; and the second from a collimated beam from the laser emitter along a path into a detector. The emitter emitted an isotropic laser light, which passed through a semi-silvered mirror, was then collimated into a beam and then directed down an adjustable length track. At the end of this was a mirror to reflect the laser down its original path, being focused by the convex lens, and now reflecting on the semi-silvered mirror into the point of a detector using an avalanche photodiode (APD) module. The light was focused to be around 5 mm in diameter on the detector to account for any small changes in position when the mirror was moved.

A Lissajous figure was considered to measure the phase difference of the two sources but it was deemed too noisy for any accurate data. Instead, the inbuilt phase-measuring function of the oscilloscopes was used. The track included a scale with a smallest division of 1 mm which provided an adequately small enough uncertainty for this experiment.

In **Method 1**, the laser frequency was set to a constant 80 MHz as with a speed of light of the order of magnitude of 3×10^8 m s^{-1} this would give a wavelength of around 4 m, which would provide a good range of phase shifts for our investigation. A zero position for L was chosen at the start of the track, coinciding with the zero position of the scale on the track, so measurements of L were distances from this position, which were easy to measure as L could be directly read from the scale on the track. The track was 2 m long, so a range of measurements from 0 m to 1.75 m was taken, each measurement 0.25 m from the last.

In **Method 2**, the track length was set to a constant 1.95 m, the longest track length available. A range of frequencies from 40 MHz to 100 MHz was chosen, with fewer frequencies chosen at the lower range as these introduced more uncertainty than the higher frequency measurements. The frequencies were set on the signal generator, and measured on the oscilloscope, as there was noticeable deviation from the suggested values on the signal generator, with frequencies generally being measured as lower by the oscilloscope.

3. RESULTS

Table I shows the results of the speed of light determined by each of the two methods.

Method	Speed of Light (m s^{-1})
Method 1	$(309 \pm 3) \times 10^6$
Method 2	$(193 \pm 7) \times 10^6$

TABLE I: Values of the speed of light and their associated uncertainties obtained from the methods listed. The methods used to calculate the uncertainties are described in Appendix I.

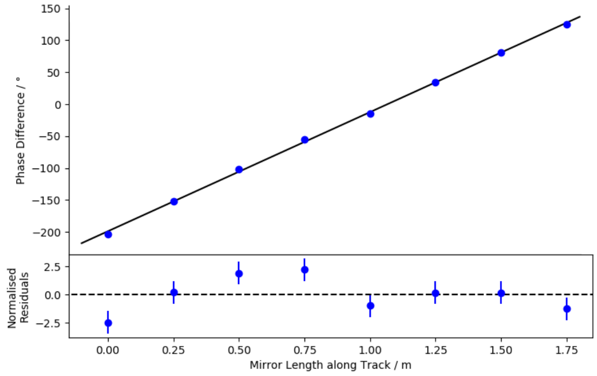

FIG. 1: The measured phase difference as a function of the track length. Trendline and error bars are added (the error bars in the top plot are too small to discern).

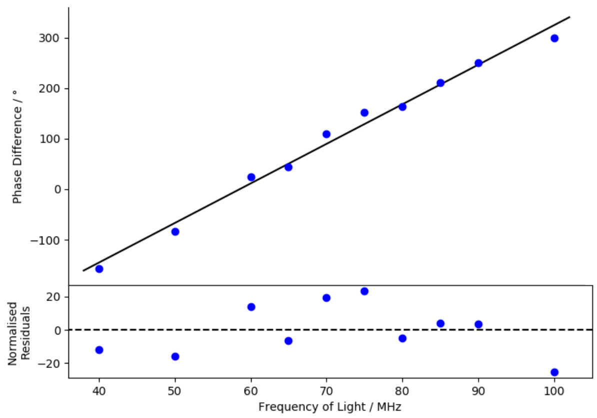

FIG. 2: The measured phase difference as a function of laser light frequency. Trendline and error bars are added (note that in both plots error bars are too small to discern).

4. DISCUSSION

In *Figure 1* the normalised residuals show that out of the 8 points, 4 of their error bars intercept the trendline, which is close to the expected 68% for a random Gaussian distribution of points, meaning the data is in reasonal agreement with a linear distribution. With more data points, we may expect this number to be closer to 68%, however there is no reason to suggest that it will converge. Also, the calculated value of intercept is -199 $\pm$ 2^o, for which an intercept of 0 is certainly out of the range. This agrees with theoretical analysis as equation [2] shows that there should be a linear relationship between phase difference and track length, with a non-zero intercept if plotted.

Method 1 resulted in a value for the speed of light which is close to the literature value of 3.00×10^8 m s^{-1} [2]. This value does not lie within the uncertainty range, but is instead three standard errors away from the calculated value. Therefore it is a poor agreement, and discrepancy likely results in an unnacounted for systematic error, such as a time delay in measuring the amplitude from the emitter or the detector, or due to differing lengths of wire.

As shown in *Figure 2*, the calculated intercept on the graph of phase difference against frequency was -8.0 $\pm$ 0.4^o, which suggests a non-zero intercept, and also as shown, the data does not fit a linear relation well, as the normalised residuals all lie far from the mean value given by the trendline. This is inconsistent with equation (3) as that suggests one should find a linear relation, with a zero intercept. Therefore, equation (3) must be inadequate to describe the situation in-hand, or there was a misunderstanding during results taking. There may be a relation between frequency and phase difference which is not apparent in equation (3), which results in a difference in relation between phase difference and frequency, and accounts for the non-zero intercept in *Figure 2*.

Method 2 resulted in a value for the speed of light that varies wildly from the accepted value of 3.00×10^8 m s^{-1}. This value is tens of standard errors from the mean, and the mean is approximately two thirds the value of the accepted value. Therefore, we believe Method 2 to be an invalid method for determining the speed of light using phase difference-relations.

Given more time for data collection, we would use **Method 1**, and collect more data points. This could verify whether equation (2) is an accurate description of the physical situation, and also reduce the uncertainty of the final calculation.

5. CONCLUSIONS

The speed of light was determined using two different methods. First by measuring the phase difference of two lasers of fixed frequency, one which had travelled an extra path difference, which was varied and measured also. Secondly it was measured by measuring the phase difference of two lasers of varying frequency, measured also, the second laser having travelled an extra fixed distance we call the path difference.

Only the first method resulted in a value which is in relative agreement with accepted values. The second method was discarded in lieu of the first, as the first method both agreed with a hypothetical relation and produced an appropriate value, unlike the second.

References

[1] H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 13$^{\text{th}}$ Ed., Pearson Addison-Wesley, San Francisco (2012).

[2] B Kees, *A Determination of the Speed of Light by the Resonant Cavity Method*, Phys. Rev, Volume 80, Issue 2, p298 (1950)

APPENDIX A: ERRORS APPENDIX

In order to convert L to 2L the uncertainty also had to be adjusted accordingly. It was done so using the equation [2],

$$\alpha_{2L} = 2\alpha_L. \tag{A1}$$

[This equation, like all of the equations included in Appendix A, is based on the error analysis formula given in I. G. Hughes and T. P. A. Hase, *Measurements and Their Uncertainties*, Oxford University Press: Oxford (2010).]

The uncertainty in the gradient of the slope of the graphs from both methods was determined by the least squares fitting method.

In **Method 1** the uncertainty on the calculated value of the speed of light, α_c, was calculated using the equation

$$\alpha_c = c\sqrt{\left(\frac{\alpha_m}{m}\right)^2 + \left(\frac{\alpha_f}{f}\right)^2}, \tag{A2}$$

where α_m is the uncertainty on the gradient (m) of the graph of phase shift (φ) against track length (L), α_f is the uncertainty on on the frequency (f), and c is the calculated value of the speed of light.

In **Method 2** the uncertainty on the calculated value of the speed of light, α_c, was calculated using the equation

$$\alpha_c = c\sqrt{\left(\frac{\alpha_m}{m}\right)^2 + \left(\frac{\alpha_\Delta}{\Delta}\right)^2}, \tag{A3}$$

where α_m is the uncertainty on the gradient (m) of the graph of phase shift (φ) against frequency (f), α_Δ is the uncertainty on on the path difference (Δ), and c is the calculated value of the speed of light.